Department of Economics University of São Paulo EAE-1234 - Public Economics Prof.: Pedro Forquesato Exercise list 2nd semester 2025

Instructions: The exercise list can be solved in group (maximum 4), but each member has to **write** and deliver the list separably. When solved in group, please note the group participants. Exercise lists "copy and pasted" will be punished as defined in the syllabus.

1. (Unemployment insurance) An economy is composed of identical individuals who earn R\$200 when working and zero otherwise. When unemployed (which happens with probability q), they earn an unemployment insurance of b reais from the government. This insurance is financed by a proportional payroll tax τ paid by employed workers. Assume that there is no other possible source of income, that agents consume all they earn and that the workers' preferences are represented by an utility function of the form

$$u(c) = c^{1/3}$$

- (a) Write the workers' expected utility as a function of q, b, and τ .
- (b) What is the government budget constraint when implementing an *actuarially fair* unemployment insurance program? Write τ as a function of benefits b and unemployment probability q.
- (c) Given this *balanced budget* policy, find the benefit level b that maximizes workers' expected utility as a function of unemployment risk q. What is the payroll tax τ that makes such a benefit possible? In this optimal policy, to how much unemployment risk are the workers exposed? Explain intuitively.
- (d) Assume that q = 5%. What is the workers' expected utility without unemployment insurance? And with the balanced budget optimal unemploymet insurance? Represent these results graphically.
- (e) Now assume that the unemployment probability q depends on the effort the workers employ while working, which depends on the desutility of losing their job (a function of b). This is an example of *moral hazard*. What is going to be the sign of dq/db? Discuss.
- 2. (Social assistance) Fatima has decided to work as a driver for an app, and she must choose how many hours per week she will dedicate to this endeavour. Fatima enjoys consumption c, but also treasures spending leisure time (l) with her family. Her preferences are represented by a utility function

$$U(c, l) = \frac{1}{2}\ln(c - 40) + \frac{1}{2}\ln(l).$$

Her wage in this new employment is R\$20 per work hour, and Fatima has a maximum of 80 hours per week that she can spend working or in leisure.

(a) What is Fatima's budget constraint? Solve for her optimal consumption and leisure choice.

- (b) Now imagine that the government introduces a negative marginal income tax (like the Earned Income Tax Credit in the US) that subsidizes Fatima's labor in R\$20 per hour. What is the new choice of consumption and leisure?
- (c) What is the direction of the income and substitution effects on the labor supply? Can we tell without any math which must be stronger?
- (d) Now consider that Fatima is beneficiary of a social assistance program (like the "Benefício de Superação da Extrema Pobreza" in previous Bolsa Família), which complements her income up to R\$500 if she receives less than that amount. What is her optimal labor supply? Does this program generate any inefficiency?
- (e) An economist proposes an Universal Basic Income (UBI) program, which gives R\$200 to Fatima as a lump-sum transfer. What is Fatima's new optimal choice of labor supply? Will she work less? Does this generate any inefficiency?
- 3. (Taxes and labor supply) Individuals decide their labor supply and consumption with preferences represented by an utility function of the type

$$U(c,l) = c - \frac{l^2}{2},$$

where c is consumption, l the labor supply (measured in hours per week), which earns a wage per weekly hour w and taxed at a proportional (labor) income tax with single rate of τ .

- (a) Write down the workers' budget constraint and solve it for the optimal (Marshallian) labor supply as a function of w and τ .
- (b) Draw the Laffer curve and find its optimum (the taxation that maximizes government revenue). Clarify the role of the mechanical and behavioral effects on establishing the curve's shape.
- (c) Now assume all income tax revenue is returned to workers as a lump-sum transfer (e.g. an universal basic income). Draw the workers' budget constraint. What is the behavioral effect of the UBI in this scenario?
- (d) Assume there are 2 individuals in our economy: the first earns R\$20 per hour of work, while the second earns R\$100 per hour. Find the Marshallian labor supply of each kind of individual given the tax rate that maximizes government revenue in (b) and the UBI in (c), as well as each worker's income after taxes and transfers.
- (e) What is the workers' indirect utility function (as a function of their wages and the tax), $V(w, \tau)$, already taking into account that the government revenue is returned to workers as a lump-sum transfer? What is the preferred tax rate for the poorer worker? (Here you can apply the *envelope theorem* to simplify the math.)

- (f) Now compute the taxable income elasticity regarding the net-of-tax rate for each worker at the optimal labor supply. Compare the result of the formula for the top of the Laffer curve seen in class with the maximizing rate in exercise 3 (d), as well as with the favorite rate for the poor worker in item (a).
- (g) What is the after-tax income and utility of each individual under the tax rate in item(b)? Find the tax rate that maximizes a Rawlsian welfare function.
- 4. (Taxation and savings) Imagine an economy with two periods, where individuals earn an income of Y = 100 in the first period and earn zero (retired) in the second. There is only one financial asset: a doge coin with zero risk and that pays 40% interest per year. Individuals have preferences that can be represented by the utility function

$$U = \ln c_1 + \ln c_2,$$

where c_t is the consumption at time $t, t \in \{1, 2\}$.

- (a) Define the intertemporal optimization problem of the individuals in this economy, and find the optimal consumption (for t = 1, 2) and savings.
- (b) Now assume the government introduces a comprehensive income tax (that burdens both labor and capital income) of $\tau = 20\%$. Solve for optimal consumption and savings.
- (c) Compare the results in (a) and (b). Does taxation distort intertemporal consumption decisions? Without explicit calculation, what are the directions of the substitution and income effects, and which is stronger?
- (d) Now find the optimal consumption and savings under a labor income tax $\tau_L = 50\%$. Does the labor tax distorts intertemporal consumption?
- 5. (Taxation and risk) Maria has an income of R\$1000, which she must decide between using for consumption or to start a business. Imagine that she can invest a share q of that income, namely R\$1000q, in her business, which might be successful and pay R\$3000q, or it might fail and pay zero, both with equal likelihood.

All potential profit plus the income not invested are immediately consumed. Maria's risk preferences are represented by von Neumann-Morgenstern preferences given by

$$U(w) = \mathbb{E}[\ln w].$$

- (a) What is the share q of her income that Maria invests in her new business?
- (b) Consider now that the government created a tax on capital income of 20%, with full tax loss offset. What is now the share of her wealth that Maria invests? Interpret.
- (c) What if there is no loss offset? Interpret.

6. (Corporate taxes) A small company in the food processing industry operates with a production function

$$Q^{S} = f(K, L) = 10\sqrt{\min\{K, L\}},$$

where K is measured in machines/year and L in worker hours. Consider that wage per hour is w = R\$500, the rental price of one machine is v and that the firm is a price-taker, with the price of the final product being P = R\$600.

The machines used for production are industrial ovens that cost R\$10,000 each, and the firm's financing cost is 10% a year. Assume for simplicity that the ovens last forever.

- (a) What is *v*? Compute the total cost function and the profit function of the firm. What is the optimal production quantity by the firm and its demand for capital and labor?
- (b) Now consider that the government implements a corporate tax of $\tau = 20\%$ over the company's revenue, with complete deduction of capital costs and wages. Does this corporate tax affect the optimal production and input use of the firm? Calculate and explain. What is the government revenue, and how does this tax relate to a tax on economic profits?
- (c) In the real world, governments rarely know the firms' true cost of financing, and deduce costs of capital at a fixed depreciation allowance. Imagine that this rate is 5%. What happens to the demand for capital and labor, quantity produced and the company's profit?
- (d) Compute the effective corporate tax rate for the taxation in item (c).